$$\int \underset{E}{e^{2x}} \cdot \underset{S}{\cos x}\, dx = I \qquad I = \begin{cases} u = e^{2x} \longrightarrow du = 2 \cdot e^{2x} dx \\ dv = \cos x\, dx \longrightarrow v = \int \cos x\, dx \\ \qquad\qquad\qquad\qquad v = \sin x \end{cases}$$

$$I = u \cdot v - \int v \cdot du$$

$$I = e^{2x} \cdot \sin x - \int \sin x \cdot 2 \cdot e^{2x} dx =$$

$$I = e^{2x} \cdot \sin x - 2 \underbrace{\boxed{\int \underset{S}{\sin x} \cdot \underset{E}{e^{2x}} dx}}_{I_1}$$

$$I_1 \begin{cases} u = e^{2x} \longrightarrow du = 2 \cdot e^{2x} dx \\ dv = \sin x\, dx \longrightarrow v = \int \sin x\, dx = -\cos x \end{cases}$$

$$I_1 = e^{2x} \cdot (-\cos x) - \int -\cos x \cdot 2 \cdot e^{2x} dx =$$

$$= -\cos x \cdot e^{2x} + 2 \underbrace{\boxed{\int \cos x \cdot e^{2x} dx}}_{I}$$

$$I_1 = -\cos x \cdot e^{2x} + 2 \cdot I$$

$$I = e^{2x} \cdot \sin x - 2 \cdot I_1$$

$$I = e^{2x} \cdot \sin x - 2 \cdot (-\cos x\, e^{2x} + 2I)$$

$$I = e^{2x} \cdot \sin x + 2 \cdot \cos x\, e^{2x} - 4I$$

$$5I = e^{2x} \cdot \sin x + 2 \cdot \cos x \cdot e^{2x}$$

$$I = \frac{e^{2x} \cdot \sin x + 2 \cdot \cos x \cdot e^{2x}}{5}$$

$$\boxed{I = \frac{e^{2x}(\sin x + 2 \cdot \cos x)}{5}}$$

$$\int \underset{E}{e^{3x}} \cdot \underset{S}{\sin x}\, dx \qquad I \qquad \begin{cases} u = e^{3x} \longrightarrow du = 3 \cdot e^{3x}\, dx \\ dv = \sin x\, dx \longrightarrow v = -\cos x \end{cases}$$

$$I = u \cdot v - \int v\, du$$

$$I = -\cos x \cdot e^{3x} - \int -\cos x \cdot 3 \cdot e^{3x}\, dx =$$

$$I = -\cos x \cdot e^{3x} + 3 \underbrace{\boxed{\int \underset{S}{\cos x}\; \underset{E}{e^{3x}}\, dx}}_{I_1}$$

$$I_1 = \begin{cases} u = e^{3x} \longrightarrow du = 3 \cdot e^{3x}\, dx \\ dv = \cos x \cdot dx \longrightarrow v = \sin x \end{cases}$$

$$I_1 = e^{3x} \cdot \sin x - \int \sin x \cdot 3 \cdot e^{3x}\, dx$$

$$= e^{3x} \cdot \sin x - 3 \underbrace{\boxed{\int \sin x \cdot e^{3x}\, dx}}_{I}$$

$$\boxed{I_1 = e^{3x} \cdot \sin x - 3 \cdot I}$$

$$I = -\cos x \cdot e^{3x} + 3 \cdot \left[ e^{3x} \cdot \sin x - 3 I \right]$$

$$I = -\cos x \cdot e^{3x} + 3 e^{3x} \cdot \sin x - 9 \cdot I$$

$$10 \cdot I = -\cos x\, e^{3x} + 3 \cdot e^{3x} \cdot \sin x.$$

$$\boxed{I = \frac{e^{3x}(-\cos x + 3 \sin x)}{10}.}$$